OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4730

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

* 4 7 3 0 *

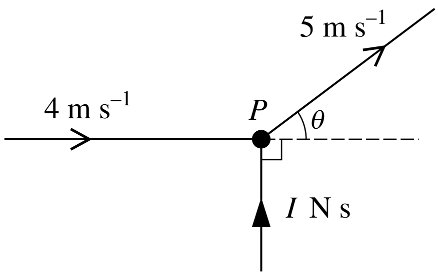
INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\ \text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

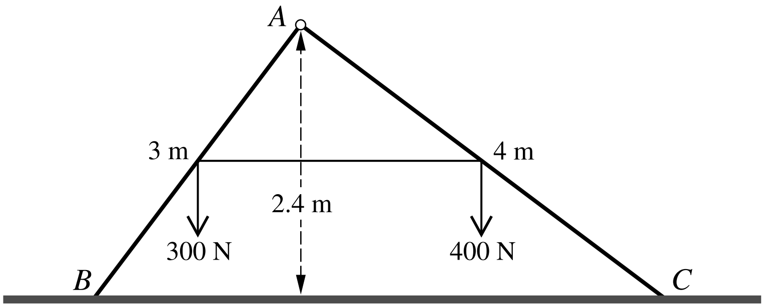
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



A particle P of mass 0.3 kg is moving in a straight line with speed 4 m s^{-1} when it is deflected through an angle θ by an impulse of magnitude I N s. The impulse acts at right angles to the initial direction of motion of P (see diagram). The speed of P immediately after the impulse acts is 5 m s^{-1}. Show that $\cos\theta = 0.8$ and find the value of I. **[4]**

2



Two uniform rods AB and AC, of lengths 3 m and 4 m respectively, have weights 300 N and 400 N respectively. The rods are freely jointed at A. The mid-points of the rods are joined by a light inextensible string. The rods are in equilibrium in a vertical plane with the string taut and B and C in contact with a smooth horizontal surface. The point A is 2.4 m above the surface (see diagram).

(i) Show that the force exerted by the surface on AB is 374 N and find the force exerted by the surface on AC. **[4]**

(ii) Find the tension in the string. **[3]**

(iii) Find the horizontal and vertical components of the force exerted on AB at A and state their directions. **[3]**

3 A particle P of mass 0.25 kg is projected horizontally with speed 5 m s^{-1} from a fixed point O on a smooth horizontal surface and moves in a straight line on the surface. The only horizontal force acting on P has magnitude $0.2v^2$ N, where v m s^{-1} is the velocity of P at time t s after it is projected from O. This force is directed towards O.

(i) Find an expression for v in terms of t. **[5]**

The particle P passes through a point X with speed 0.2 m s^{-1}.

(ii) Find the average speed of P for its motion between O and X. **[5]**

4 One end of a light inextensible string of length 2 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is held at rest with the string taut so that OP makes an angle of 0.15 radians with the downward vertical. P is released and t seconds afterwards OP makes an angle of θ radians with the downward vertical.

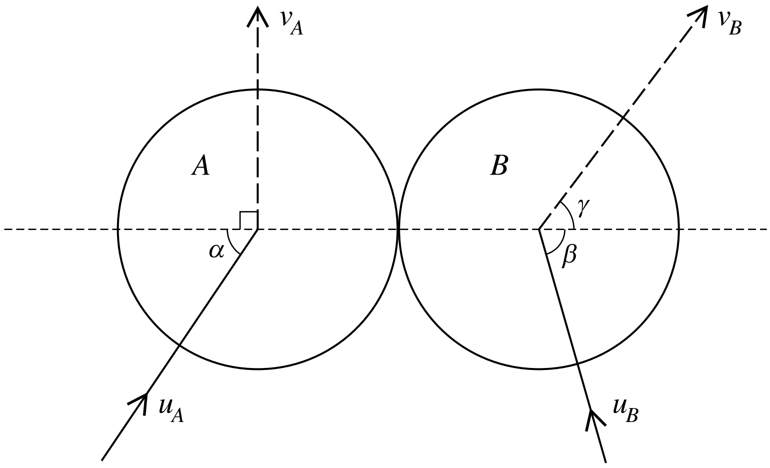
(i) Show that $\frac{d^2\theta}{dt^2} = -4.9 \sin\theta$ and give a reason why the motion is approximately simple harmonic. **[3]**

Using the simple harmonic approximation,

(ii) obtain an expression for θ in terms of t and hence find the values of t at the first and second occasions when $\theta = -0.1$, **[5]**

(iii) find the angular speed of OP and the linear speed of P when $t = 0.5$. **[3]**

5



Two uniform smooth identical spheres A and B are moving towards each other on a horizontal surface when they collide. Immediately before the collision A and B are moving with speeds u_A m s^{-1} and u_B m s^{-1} respectively, at acute angles α and β, respectively, to the line of centres. Immediately after the collision A and B are moving with speeds v_A m s^{-1} and v_B m s^{-1} respectively, at right angles and at acute angle γ, respectively, to the line of centres (see diagram).

(i) Given that $\sin\beta = 0.96$ and $\frac{v_B}{u_B} = 1.2$, find the value of $\sin\gamma$. **[2]**

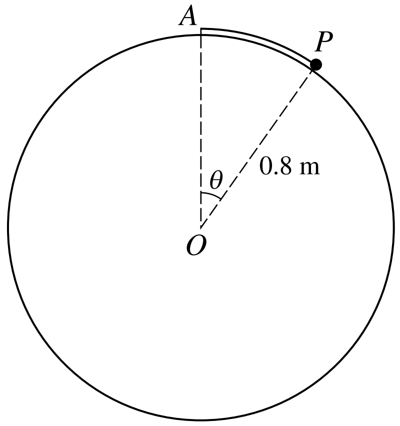
(ii) Given also that, before the collision, the component of A's velocity parallel to the line of centres is 2 m s^{-1}, find the values of u_B and v_B. **[5]**

(iii) Find the coefficient of restitution between the spheres. **[3]**

(iv) Given that the kinetic energy of A immediately before the collision is $6.5m$ J, where m kg is the mass of A, find the value of v_A. **[2]**

[Questions 6 and 7 are printed overleaf.]

6



A particle P of weight 6 N is attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 0.8 m. The string has natural length $\frac{1}{10}\pi$ m and modulus of elasticity 9 N. P is released from rest at a point X on the sphere where OX makes an angle of $\frac{1}{4}\pi$ radians with the upwards vertical. P remains in contact with the sphere as it moves upwards to A. At time t seconds after the release, OP makes an angle of θ radians with the upwards vertical (see diagram). When $\theta = \frac{1}{6}\pi$, P passes through the point Y.

(i) Show that as P moves from X to Y its gravitational potential energy increases by $2.4(\sqrt{3} - \sqrt{2})$ J and the elastic potential energy in the string decreases by 0.4π J. **[5]**

(ii) Verify that the transverse acceleration of P is zero when $\theta = \frac{1}{6}\pi$, and hence find the maximum speed of P. **[6]**

7 One end of a light inextensible string of length 0.8 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. P is projected horizontally from the point 0.8 m vertically below O with speed 5.6 m s^{-1}. P starts to move in a vertical circle with centre O. The speed of P is v m s^{-1} when the string makes an angle θ with the downward vertical.

(i) While the string remains taut, show that $v^2 = 15.68(1 + \cos\theta)$, and find the tension in the string in terms of θ. **[7]**

(ii) For the instant when the string becomes slack, find the value of θ and the value of v. **[3]**

(iii) Find, in either order, the speed of P when it is at its greatest height after the string becomes slack, and the greatest height reached by P above its point of projection. **[4]**

1	 [5cos θ − 4 = 0] cos θ = 0.8 [I = 0.3(5sin θ - 0) or sin θ = $I \div$ (0.3 x 5)] I = 0.9	 M1 A1 M1 A1 [4]	For using $v_x - u_x = 0$ **or** for a triangle sketched with sides I/0.3, 4 and 5 with angles θ and 90° opposite I/m and 5 respectively. AG For using I = $m\,(\Delta v)$ in 'y' direction or $I = \sqrt{\left((0.3\times5)^2 - (0.3\times4)^2\right)}$ M1

2 i	 (1.8 + 3.2)R_B = (3.2 + 0.9)x300 + 1.6x400 Force exerted on AB is 374 N Force exerted on AC is 326 N	M1 A1 A1 B1 [4]	For taking moments about C for the whole for M1 need 3 terms; allow 1 sign error and/or 1 length error and/or still including sin/cos or for taking moments about B for whole (1.8 + 3.2)R_C = (1.8 + 1.6)x400 + 0.9x300 giving force on AC first: M1A1A1A1
ii	 0.9x300 + 1.2T = 1.8x374 Tension is 336 N	M1 A1 A1 [3]	For taking moments about A for AB for M1 need 3 terms, allow 1 sign error and/or 1 length error and/or still including sin/cos or moments about A for AC 1.6x400 + 1.2T = 3.2x326
iii	Horizontal component is 336 N to the left [Y = 374 – 300] Vertical component is 74 N downwards	B1ft M1 A1ft [3]	 For resolving forces on AB vertically

Give credit for part (ii) done on the way to part (i) if not contradicted in (ii).

3 i	$0.25(dv/dt) = -0.2v^2$	M1	For using Newton's second law with $a = dv/dt$. Allow sign error and/or omitting mass
	$0.25\int v^{-2}dv = -0.2t(+C)$	dep M1	For separating variables and attempting to integrate (ie get v^{-1} and t).
	$-v^{-1}/4 = -t/5 + C$	A1	
	$[1/4v = t/5 + 1/20]$	M1	For using $v(0) = 5$ to obtain C
	$v = \dfrac{5}{4t+1}$ oe	A1 [5]	
ii		M1	For using $v = dx/dt$ and integrating
	$x = (5/4)\ln(4t + 1)$ $(+ B)$	A1	
	Subst $v = 0.2$ in (i) to find t	M1	Implied by $t = 6$
	Obtain $x(6)$ ($= 1.25 \ln 25$ oe (4.02359...))	M1	
	Average speed is 0.671 ms^{-1}	A1 [5]	May be written as $\frac{5}{12}\ln 5$
	Alternatively		
		M1	For using $mv(dv/dx) = -0.2v^2$, separating variables and integrating. Allow sign error and/or omitting mass.
	$\ln v = -0.8x + B$	A1	
	Subst $v = 0.2$ in (i) to find t	M1	Implied by $t = 6$
	Obtain $x(0.2)$ (= $1.25 \ln(5/0.2)$ oe (4.0239...))	M1	
	Average speed is 0.671 ms^{-1}	A1	May be written as $\frac{5}{12}\ln 5$

4 i	$[-0.2\text{x}2\,\ddot{\theta} = 0.2g\sin\theta]$	M1	For using Newton's second law transversely. Allow sign error and/or sin/cos error and/or missing 0.2, g or l.
	$\dfrac{d^2\theta}{dt^2} = -4.9\sin\theta$	A1	AG
	For small θ, $\sin\theta \approx \theta$ **and** $\ddot{\theta} = -4.9\theta$ represents SHM	B1 [3]	
ii		M1	For using $\theta = A\cos(nt)$ or $A\sin(nt + \varepsilon)$. Allow sin/cos confusion
	$\theta = 0.15\cos(\sqrt{4.9}\,t)$ oe	A1	
	$t = 1.04$ at first occasion	A1	
		M1	for using $t_1 + t_2 = 2\pi / n$
	$t = 1.80$ at second occasion	A1 [5]	
iii		M1	For using $\dot{\theta} = -An\sin(nt)$ oe. Allow sign error and/or ft from θ in (ii).
	Angular speed is (-) 0.297 rads s^{-1}	A1	
	Linear speed is (-) 0.594ms^{-1}	A1ft [3]	

In (ii) & (iii) allow M marks if angular displacement/speed has been confused with linear.

5 i	[$\sin\gamma = 0.96 \div 1.2$] $\sin\gamma = 0.8$	M1 A1 [2]	For using $v_B\sin\gamma = u_B\sin\beta$
ii	$(m)2 - (m)u_B\cos\beta = (m)v_B\cos\gamma$ $2 = v_B(0.6 + 0.28 \div 1.2)$ $v_B = 2.4$, $u_B = 2$	M1 A1 M1 A1 A1 [5]	For using the principle of conservation of momentum. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. allow $u_A\cos\alpha$ (instead of 2) for A1 For eliminating u_B or v_B. Allow with cos Or $2 = 0.28u_B + 0.72u_B$
iii	[$(2 + u_B\cos\beta)e = v_B\cos\gamma$] $(2 + 2 \times 0.28)e = 2.4 \times 0.6$ $e = \frac{9}{16}$ or 0.5625	M1 A1ft A1 [3]	For applying Newton's exp'tal law. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. ft u_B and v_B only
iv	[$(y\text{-component})^2 = 13 - 4$] $v_A = (y\text{-component})_{\text{before}} = 3$	M1 A1 [2]	For using $\frac{1}{2}(m)v^2 = 6.5(m)$ **and** $(y\text{-component})^2 = v^2 - 2^2$. Allow 1 slip.

6 i	PE gain = $6 \times 0.8(\sqrt{3}/2 - 1/\sqrt{2})$ $= 2.4(\sqrt{3} - \sqrt{2})$ EE loss = $\frac{9}{2(\pi/10)}[(0.8\pi/4 - \pi/10)^2 - (0.8\pi/6 - \pi/10)^2]$ EE loss = $45\pi[(0.2 - 0.1)^2 - (0.4 - 0.3)^2 \div 9]$ $= 5\pi(9 \times 0.01 - 0.01) = 40\pi/100 = 0.4\pi$ J	M1 A1 M1 A1 A1 [5]	For using PE gain = $W(h_Y - h_X)$ Shown fully, with no slips AG For using EE loss = $\lambda(e_X^2 - e_Y^2)/2l$. Allow slips for M1. Fully correct No slips in simplification AG
ii	$T = 9(0.8\pi/6 - \pi/10) \div (\pi/10)$ $W\sin\theta - T = 6 \times \sin(\pi/6) - 90 \times (0.2 \div 6) = 0$ ➔ transverse acceleration is zero $\frac{1}{2}(6/9.8)v^2 = 0.4\pi - 2.4(\sqrt{3} - \sqrt{2})$ Maximum speed is 1.27 ms^{-1}	B1 M1 A1 M1 A1 A1 [6]	 For attempting to show that $W\sin\theta - T = 0$ at Y by subst $\theta = \pi/6$ AG No slips For using KE gain = EE loss – PE gain at Y. Need 3 terms, allow sign errors and/or g omitted.

7 i		M1	For using the principle of conservation of energy. Allow sign error, sin/cos; need 3 terms.
	$\frac{1}{2} mv^2 = \frac{1}{2} m5.6^2 - mg0.8(1 - \cos\theta)$	A1	
	$v^2 = 15.68(1 + \cos\theta)$	A1	AG No slips
	$T - mg\cos\theta = mv^2/r$	M1 A1	For using Newton's second law. Allow sign error and/or sin/cos and/or m omitted
	$[T - 0.3g\cos\theta = 0.3\text{x}15.68(1 + \cos\theta)/0.8]$	M1	For substituting for v^2
	Tension is $2.94(3\cos\theta + 2)$ N oe	A1 [7]	
ii		M1	For putting $T = 0$ and attempting to solve
	θ is 131.8° (or 2.3 rads) Accept 132° (exact)	A1	accept $\theta = \cos^{-1}(-2/3)$
	v is 2.29	B1 [3]	$\sqrt{15.68/3}$ exact
iii	[speed = $\lvert v\cos(180 - \theta)\rvert = \sqrt{15.68/3} \times (2/3)$]	M1	For using 'speed at max. height = horiz. comp. of vel. when string becomes slack'
	Speed at greatest height is 1.52 ms^{-1}	A1	
	$0.3gH = \frac{1}{2}\, 0.3(5.6^2 - 1.52...^2)$	M1	For using the principle of conservation of energy
	Greatest height is 1.48 m	A1 [4]	40/27 exact
	ALTERNATIVE for (iii)		
	$[0 = 2.286..^2 \times (1\text{-}4/9) - 19.6y$, $H = 0.8(1 + 2/3) + y]$ $H = 1.3333.. + 0.1481...\ (4/3 + 4/27)$	M1	For using $0^2 = \dot{y}^2 - 2gy$ and $H = 0.8\{1 + \cos(180 - \theta)\} + y$
	Greatest height is 1.48 m (40/27)	A1	
	$[\frac{1}{2} m(2.286...^2 - \text{speed}^2) = mg \times 0.1481....$ $\text{speed}^2 = 2.286..^2 - 19.6 \times 0.1481....]$ or $[\frac{1}{2} m(5.6^2 - \text{speed}^2) = mg \times 1.481....$ $\text{speed}^2 = 5.6^2 - 19.6 \times 1.481....]$	M1	For using the principle of conservation of energy
	Speed at greatest height is 1.52 ms^{-1}	A1	